TOKYO J. MATH.
VOL. 11, NO. 2, 1988

On Peak Sets for Certain Function Spaces

Shinya YAMAGUCHI and Junzo WADA

Waseda University

Introduction.

Let A be a function space on a compact Hausdorff space X. In this paper, we show that some theorems on function algebras can be generalized to the case of function spaces A having certain conditions. E. Briem [2] proved the following: Let A be a function algebra. If any peak set for the real part $\mathrm{Re}\,A$ of A is a peak set for A, then $A=C(X)$, where $C(X)$ denotes the Banach algebra of complex-valued continuous functions on X with the supremum norm. In association with the theorem of Briem, we consider the class of function spaces having the condition (A) (see §1). It is a wider class containing the class of function algebras. We here discuss whether theorems on function algebras can be generalized to the case of the class.

In §1, the Bishop antisymmetric decomposition theorem for function spaces is given. This is a generalization of Bishop's theorem [1] on function algebras. In §2 we give some examples of function spaces having (A). In §3 we consider the class $\mathscr{A}$ of function spaces having (A) and give characterizations to assert that $A=C(X)$ for $A\in\mathscr{A}$. These results are generalizations of theorems on function algebras.

§1. Bishop antisymmetric decomposition for function spaces.

Throughout this parer, X will denote a compact Hausdorff space. A is said to be a *function space* (resp. *function algebra*) on X if A is a closed subspace (resp. subalgebra) in $C(X)$ containing constant functions and separating points in X.

Let A be a function space on X. For a subset E in X, we denote

$$A(E)=\{f\in C(E):\ fg\in A|_E \text{ for any } g\in A|_E\}\ ,$$
$$A_R(E)=\{f\in C_R(E):\ fg\in A|_E \text{ for any } g\in A|_E\}$$

Received January 18, 1988