

On Sullivan's Vanishing Cycles in Codimension-One Foliations

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(Communicated by S. Suzuki)

Introduction.

A Sullivan's vanishing cycle is a cycle in a leaf of a foliation which is in a sense essential in the leaf and inessential if it is displaced to nearby leaves. This notion is first introduced by Novikov in his celebrated work [N] in order to prove his closed leaf theorem. In [Su] Sullivan defined a notion of higher dimensional vanishing cycles in foliations of arbitrary codimensions which includes Novikov's one as 1-dimensional case. Sullivan showed that a Sullivan's vanishing cycle yields a non-trivial foliation cycle and gave an alternative proof of Novikov's closed leaf theorem for S^3 . In the previous paper [Miy] we gave a sufficient condition for the existence of Sullivan's vanishing cycles in codimension-one foliations and also showed a closed leaf theorem which in a sense generalizes Novikov's closed leaf theorem to the higher dimensional case.

In the present paper we will study topological aspects of codimension-one foliations which the existence of Sullivan's vanishing cycles yields. Our main theorem (Theorem B) gives some necessary and sufficient conditions for the existence of a Sullivan's vanishing cycle in case that there is no Novikov's vanishing cycle. Although Theorem A is the main theorem of [Miy], the proof in [Miy] is rather sketchy and it plays a key role in the proof of Theorem B. Therefore for the completeness we will give a complete proof of Theorem A which is also improved compared with the one given in [Miy]. Theorem C asserts that a Novikov's vanishing cycle yields a higher dimensional singular manifold chain bounded by a tangential cycle, which is the partial converse of Theorem A.

Contents are as follows: In Section 1 we state the results. We study a pull back of a foliation without Sullivan's vanishing cycles in Section 2 and by applying the results in part we study a foliation without