

Logarithmic Fano Manifolds Are Simply Connected

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§1. Introduction.

The purpose of this short note is to show that the branched covering method in [6] is also effective in the case of positive first Chern class. Hence there is nothing new from the technical point of view. But it will be worthwhile to point out that even in differential geometry it is useful to study smooth differentiable manifolds through orbifold structures.

Let X be a smooth projective algebraic variety over C . X is said to be a Fano manifold if the canonical bundle K_X is negative. By the solution of Calabi's conjecture ([7]), X is Fano if and only if X admits a Kähler metric of positive Ricci curvature. Then by using Myer's theorem ([4]) and the Kodaira vanishing theorem, one can easily see that every Fano manifold is simply connected (cf. [1]).

Let X be a smooth projective algebraic variety over C and let D be a divisor on X with simple normal crossings. The pair (X, D) is said to be a logarithmic Fano manifold if the logarithmic canonical bundle $K_X + D$ is negative. This is a natural generalization of the notion of Fano manifolds.

In this paper, we prove the following theorem.

THEOREM 1. *Let (X, D) be a logarithmic Fano manifold. Then X is simply connected.*

The essential point of the proof is to see that X has an orbifold structure with a Kähler metric of positive Ricci tensor. The rest of the proof is a minor modification of the argument in [1].

REMARK 1. It is plausible that if a projective manifold X over C satisfies that $\kappa(-K_X) = \dim X$, then X is simply connected. Theorem 1 is a partial answer to this problem.