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Weak Asymptotical Stability of Yang-Mills' Gradient Flow

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Introduction.

Let $J(\cdot)$ be a functional on some functional space X, and $u_0 \in X$ be a critical point of $J(\cdot)$, i.e., the solution of the variational problem

$$\operatorname{grad} J(u_0)=0 ,$$

where $-\operatorname{grad} J(\cdot)$ is the Euler-Lagrangian operator of $J(\cdot)$.

Concerning the variational problems, there are two important problems, the existence of critical points and their stability.

The classical Morse theory covers the analysis of the variational problems on finite-dimensional spaces. In differential geometry, we find several variational problems on infinite-dimensional spaces. For such problems in discussing the properties of a critical point, several authors study those of the corresponding gradient flow. The *gradient flow* $u(t)$ of $J(\cdot)$ with the initial value v is, if exists, a C^1-flow satisfying

$$\begin{cases} \dfrac{du(t)}{dt}=-\operatorname{grad} J(u(t)) & t \in (0, \infty) , \\ u(0)=v . \end{cases}$$

A typical variational problem in differential geometry is the harmonic map problem, i.e., that of critical maps of the energy integral defined on maps $f: M \to N$ between two Riemannian manifolds:

$$J(f)=E(f)=\frac{1}{2}\int_M |df|^2 *1 .$$

In 1964 Eells and Sampson showed the existence of harmonic maps by use of the gradient flow corresponding to the energy integral (the Eells-Sampson equation) in [2]. Recently, Naito [13] has clarified the

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