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Complete Space-Like Surfaces with Constant Mean Curvature in the Minkowski 3-Space

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Introduction.

Let L^3 be the Minkowski 3-space, that is, $\boldsymbol{R}^3$ with the indefinite metric $\langle\ ,\ \rangle=(dx^1)^2+(dx^2)^2-(dx^3)^2$. A surface in L^3 is called *space-like* if the induced metric on the surface is positive definite. On a space-like surface, the notions of the first fundamental form, the second fundamental form, and the mean curvature are defined in the same way as on a surface in the euclidean space.

In particular, we shall consider complete space-like surfaces with constant mean curvature H. For example, in [2] and [4], Calabi and Cheng-Yau established the Bernstein-type theorem when $H\equiv 0$, *maximal* space-like surface. In other words, the uniqueness theorem holds for maximal surfaces.

In this paper, we investigate complete space-like surfaces with *non-zero* constant mean curvature H. In this case, uniqueness does not hold and there are several examples. The most well-known example of such a surface is the *pseudosphere*:

$$S(H)=\left\{(x^1,\ x^2,\ x^3)\in L^3\ ;\ (x^1)^2+(x^2)^2-(x^3)^2=-\frac{1}{H^2},\ x^3>0\right\}\ , \tag{0.1}$$

which is the only complete, totally umbilical space-like surface with constant mean curvature H. Note that $S(H)$ is isometric to the Poincaré disc with constant Gaussian curvature $-H^2$.

Among non-umbilical space-like surfaces, the following *hyperbolic cylinder* is the simplest one:

$$C(H)=\left\{(x^1,\ x^2,\ x^3)\in L^3\ ;\ (x^1)^2-(x^3)^2=-\frac{1}{4H^2},\ x^3>0\right\}\ . \tag{0.2}$$

This is the only complete, flat space-like surface with non-zero constant mean curvature H.

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