TOKYO J. MATH.
VOL. 11, NO. 2, 1988

A Non-Existence Result for Harmonic Mappings from $\boldsymbol{R}^n$ into $\boldsymbol{H}^n$

Atsushi TACHIKAWA

Keio University

(Communicated by M. Obata)

§0. Introduction.

The purpose of this paper is to give a non-existence result for harmonic mappings defined on the whole $\boldsymbol{R}^n$, a Euclidean n-space ($n \geqq 2$), into a real hyperbolic n-space $\boldsymbol{H}^n$.

For harmonic mappings $U: M \to N$ (M, N: complete Riemannian manifolds) some Liouville type theorems have been proved. By S. Hildebrandt - J. Jost - K.-O. Widman [4] it has been shown that a harmonic mapping $U: M \to N$ must be a constant mapping if M is *simple* and image $U(M)$ is contained in a geodesic ball $B_R(Q) \subset N$ with $R < \pi/(2\sqrt{\kappa})$ where κ denotes the maximum of the sectional curvatures of N. Here, a Riemannian manifold is said to be *simple*, if it is topologically $\boldsymbol{R}^m$ furnished with a metric for which the associated Laplace-Beltrami operator is uniformly elliptic on $\boldsymbol{R}^m$. (See also [1] and [6].) Moreover by L. Karp [5] it has been shown that, for a complete, noncompact Riemannian manifold M and a simply-connected Riemannian manifold N with nonpositive sectional curvature, a nonconstant harmonic mapping $U: M \to N$ satisfies a certain growth-order condition. This implies a non-existence theorem for harmonic mappings under some growth condition. On the contrary, our non-existence theorem in this paper requires no growth condition.

In order to describe our main result precisely we introduce some notations: We use a standard coordinate system $x = (x^1, \cdots, x^n)$ on $\boldsymbol{R}^n$ and a normal coordinate system $u = (u^1, \cdots, u^n)$ centered at some point P_0 on $\boldsymbol{H}^n$. $\langle \cdot, \cdot \rangle$ and $|\cdot|$ stand for the Euclidean scalar product and norm. We shall write $(g_{ij}(u))$ for the metric tensor on $\boldsymbol{H}^n$ with respect to the normal coordinate system $(u^i)_{1 \leq i \leq n}$, $(g^{ij}(u))$ for the inverse of $(g_{ij}(u))$, and the Christoffel symbols of the first and second kind of the Levi-Civita connection on $\boldsymbol{H}^n$ will be denoted by Γ_{ijk} and Γ^i_{jk}.

A mapping $U: \boldsymbol{R}^n \to \boldsymbol{H}^n$ is said to be a *harmonic mapping* if it is of

Received August 26, 1987