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A Note on Ideal Bases

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§ 1. Construction of ideal bases.

In Okutsu [2], integral bases of finite separable extensions of quotient fields of Dedekind domains were obtained in terms of "divisor polynomials" introduced there. The purpose of this note is to generalize this result to ideal bases. In § 2, we shall give an example which illustrates this result.

Let $\mathfrak{o}$ be a Dedekind domain with the quotient field k and $f(x)$ a monic irreducible separable polynomial of degree n in $\mathfrak{o}[x]$. Let θ be one of the roots of $f(x)$ in an algebraic closure $\bar{k}$ of k. Let $K=k(\theta)$ and assume that $\mathfrak{A}$ is a fixed non-zero ideal of K throughout this section. Let $\{\mathfrak{p}_\lambda\}_{\lambda\in\Lambda}$ be the set of all prime ideals of $\mathfrak{o}$, k_λ a completion of k with respect to $\mathfrak{p}_\lambda$, and $\mathfrak{o}_\lambda$ its valuation ring. Fixing an embedding of k in k_λ, we assume that k is a subfield of k_λ. Let $\bar{k}_\lambda$ be an algebraic closure of k_λ. We denote by $\varphi_\lambda(\ )$ (or $\varphi_{\mathfrak{p}_\lambda}(\ )$) the exponential valuation on $\bar{k}_\lambda$ which is the unique extension of the $\mathfrak{p}_\lambda$-adic valuation of k. For each $\lambda\in\Lambda$, let $f(x)=\prod_{i=1}^{s_\lambda} f_{\lambda,i}(x)$ be a factorization of $f(x)$ in $k_\lambda[x]$, where $f_{\lambda,i}(x)$ is a monic irreducible polynomial in $\mathfrak{o}_\lambda[x]$. For simplicity, we will write s for s_λ in some cases. Let $\theta_{\lambda,i}$ be one of the roots of $f_{\lambda,i}(x)$ in $\bar{k}_\lambda$. We define a k-isomorphism $\iota_{\lambda,i}$ from $K=k(\theta)$ into $\bar{k}_\lambda$ by putting $\iota_{\lambda,i}(\theta)=\theta_{\lambda,i}$. Let $\iota_{\lambda,i}(\mathfrak{A})=\mathfrak{A}\mathfrak{o}_{k_\lambda(\theta_{\lambda,i})}$ which is an ideal of $k_\lambda(\theta_{\lambda,i})$ where $\mathfrak{o}_{k_\lambda(\theta_{\lambda,i})}$ is the valuation ring of $k_\lambda(\theta_{\lambda,i})$. For each $\lambda\in\Lambda$, we define a rational-valued and ∞-valued function $\Phi_\lambda(\ )$ on K as follows:

$$\Phi_\lambda(\alpha)=\min_{1\le i\le s_\lambda}\{\varphi_\lambda(\iota_{\lambda,i}(\alpha))-\varphi_\lambda(\iota_{\lambda,i}(\mathfrak{A}))\}\qquad(\alpha\in K)\,.$$

Then we note that α is an element of $\mathfrak{A}$ if and only if $\Phi_\lambda(\alpha)\geqq 0$ for any $\lambda\in\Lambda$. For a polynomial $g(x)=a_0x^m+\cdots+a_m$ in $\mathfrak{o}[x]$, we put

$$\varphi_\lambda(g(x))=\min_{0\le j\le m}\varphi_\lambda(a_j)\,.$$

PROPOSITION 1. *For each $\lambda\in\Lambda$ and any positive integer m ($<n$), there*

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