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A Sum Formula for Casson's λ-Invariant

Shinji FUKUHARA and Noriko MARUYAMA

Tsuda College

Dedicated to Professor Itiro Tamura on his 60th birthday

A. Casson [1] defined an integer valued invariant $\lambda(M)$ for an oriented homology 3-sphere M.

In [4] J. Hoste gave a formula to calculate $\lambda(M)$ from a special framed link description of M. He required the framed link to satisfy the condition that linking numbers of any two components of the link are zero.

In this note, we give a sum formula to calculate Casson's λ-invariant for an oriented homology 3-sphere which is constructed by gluing two knot exteriors in homology 3-spheres with some diffeomorphism between their boundaries. Our result is just the λ-invariant version of C. Gordon's theorem [2, Theorem 2] for μ-invariant.

§1. Preliminaries.

Casson proved the following theorem.

THEOREM 1 (Casson). *Let M be an oriented homology 3-sphere. There exists an integer valued invariant $\lambda(M)$ with the following properties.*

(1) *If $\pi_1(M)=1$, then $\lambda(M)=0$.*

(2) $\lambda(-M)=-\lambda(M)$, *where $-M$ denotes M with the opposite orientation.*

(3) *Let K be a knot in M and $(K_n; M)$ be the oriented homology 3-sphere obtained by performing $1/n$-Dehn surgery on M along K, $n\in \boldsymbol{Z}$. $\lambda(K_{n+1}; M)-\lambda(K_n; M)$ is determined independently of n.*

(4) $\lambda(M)$ *reduces,* mod 2, *to the Rohlin invariant $\mu(M)$.*

By the property (3), $\lambda'(K; M)=\lambda(K_{n+1}; M)-\lambda(K_n; M)$ is well defined. By the induction on n, we have:

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