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Lipschitz Classes and Fourier Series of Stochastic Processes

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§1. Introduction.

Let $f(t)\in L^1(T)$, $T=(-\pi, \pi)$, be a 2π-periodic function and write, for a positive integer j,

$$\Delta_h^{(j)}f(t)=\sum_{k=0}^{j}(-1)^{j-k}\binom{j}{k}f(t+kh)\ , \tag{1.1}$$

$$L^{(j)}(h, t; f)=h^{-1}\int_0^h \Delta_u^{(j)}f(t)du\ . \tag{1.2}$$

Kinukawa [4] has discussed the problem to characterize the Lipschitz class of $f(t)$ satisfying

$$ {}_aA_{p,j,\alpha}^{\circ}(f)=\left(\int_0^1 h^{-1}dh\left\{\int_T [h^{-\alpha}|\Delta_h^{(j)}f(t)|]^a dt\right\}^{p/a}\right)^{1/p}<\infty\ , \qquad (\alpha, a, p>0) \tag{1.3}$$

in terms of Fourier coefficients of the functions of the class. He also discussed a more general class of $f(t)$ for which

$$ {}_aA_{p,j,\alpha}(f)=\left(\int_0^1 h^{-1}dh\left\{\int_T [h^{-\alpha}|L^{(j)}(h, t; f)|]^a dt\right\}^{p/a}\right)^{1/p}<\infty\ , \tag{1.4}$$

generalizing a Yadav's result on absolute convergence of Fourier series.

We are interested in a more general Lipschitz class for a later purpose.

Throughout this paper, $\phi(t)$ is either identically one on $[0, 1]$ or a nonnegative nondecreasing function such that $\phi(0)=0$ and $t^{-1}\phi(t)$ is nonincreasing on $(0, 1]$.

We introduce, for a nonnegative integer l,

$$ {}_aA_{p,j,\alpha}^{l,\phi}(f)=\left(\int_0^1 h^{-l-1}[\phi(h)]^{-1}dh\left\{\int_T [h^{-\alpha}|L^{(j)}(h, t; f)|]^a dt\right\}^{p/a}\right)^{1/p} \qquad (\alpha, a, p>0)\ . \tag{1.5}$$

Our main purpose is to study on the class of stochastic processes which

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