TOKYO J. MATH.
VOL. 11, No. 2, 1988

Strongly Stable Factors, Crossed Products and Property Γ

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(Communicated by K. Ogiue)

§ 1. Introduction.

The important problems in the theory of factors of type II_1 are how to construct factors of type II_1 and classify them. As methods of their constructions, we know the operations of taking crossed product and tensor product. As a method of their rough classification, we know the method by using centralizing sequences. When we classify them by using centralizing sequences, first of all, we have two classes: one is factors with property Γ and the other is full factors. The class with property Γ contains an important subclass of factors, which are said to be strongly stable. For instance, the hyperfinite factor of type II_1 is strongly stable. On the other hand, the group von Neumann algebra associated with the free group on 2 generators is full.

In general, it is a basic problem whether a structure property of a von Neumann algebra is compatible with the operations of taking crossed product and tensor product or not. For instance, it is known [11] and is an important result that the crossed product of the hyperfinite factor of type II_1 by a free action of an amenable group is also hyperfinite. In the first part of the present paper (§ 3), we treat the class of strongly stable factors from the above point of view. We shall show (Theorem 3.1 and Corollary 3.2) that the crossed product of a strongly stable factor by a free action of an amenable group is again strongly stable. We shall then consider, as a converse of this result, the problem under what conditions the assumption the crossed product $M\times_\alpha G$ of an action α of a group G being strongly stable implies that the original algebra M is strongly stable. The property of G for the case is non inner amenable (Proposition 3.3). Moreover we shall refer to the operations of tensor product and show that if a tensor product $M\otimes R$ between factors of type II_1 is strongly stable and R has Connes and Jones' property T, then M

Received July 22, 1987