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K-Groups and λ-Invariants of Algebraic Number Fields

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Dedicated to the late Professor Suguru Hamada

Introduction.

Let F be a totally real algebraic number field, O_F the integer ring of F and $K_m(O_F)$ Quillen's higher K-group of O_F for each non-negative integer m. According to Quillen [8], $K_m(O_F)$ is a finite abelian group for even $m=2n$ $(n \geqq 1)$. Let p be an odd prime number and F' a Galois p-extension of F. In this paper, we investigate whether the prime p divides the order of $K_{2n}(O_{F'})$. (The order of $K_2(O_{F'})$ has been treated by several authors [2], [4], [9].) We shall state our main theorem in §1. In §2, we prove group-theoretical lemmas on $\boldsymbol{Z}_p$-modules on which a finite group acts, whose order is prime to p.

In the final part §3, we prove our main theorem in using first a result of Soulé, according to which we translate the language of K-theory into that of Iwasawa theory, then a result of Iwasawa (Lemma 4), with the help of which we refine Kida's formula (Lemma 5), which leads immediately to our theorem.

§1. Main theorem.

Throughout the following, let p be a fixed odd prime number. For a finite algebraic number field F, we denote by F_∞ the cyclotomic $\boldsymbol{Z}_p$-extension of F.

THEOREM. *Let F be a totally real algebraic number field of finite degree, F' a Galois p-extension of F, ζ a primitive p-th root of* 1 *and n an odd positive integer. Let k denote $F(\zeta)$ and d the degree $(k:F)$. We assume that the μ-invariant μ_k of k_∞/k is zero. Then we have the*

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