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Fourier Series with Nonnegative Coefficients on Compact Semisimple Lie Groups

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§1. Introduction.

Let G be a compact abelian group and $G^\wedge$ the dual of the group G. For f in $L^1(G)$, $f^\wedge$ denotes the Fourier transform of f. Then it is well known that functions in $L^1(G)$ with positive Fourier coefficients that are pth $(1<p\leqq 2)$ power integrable near the identity in G have Fourier coefficients in l^q, where $q=p/(p-1)$. When $p=2$, this result was proved by N. Wiener for $G=T$, the circle group, (cf. [B]) and by M. Rains for compact abelian groups (see [R]). For $1<p<2$ it was shown by J. M. Ash, M. Rains and S. Vági (see [ARV]). Recently, H. Miyazaki proved that the same result also holds for central functions on $SU(2)$ (see [M]). In this paper, applying the technique used in [ARV], we shall prove that the similar result holds for central and zonal functions on compact semisimple Lie groups.

When G is a compact abelian group, the characters χ_α $(\alpha\in G^\wedge)$ satisfy $\chi_\alpha\chi_\beta=\chi_{\alpha+\beta}$ $(\alpha, \beta\in G^\wedge)$, and thus, $(fg)^\wedge=f^\wedge * g^\wedge$; this property plays an important role in the proof of [ARV]. However, when G is an arbitrary compact group, the characters and the spherical functions on G don't satisfy such a simple formula; actually, the Clebsch-Gordan formula for characters and the addition formula for spherical functions offer the replacement. Then applying the same argument in [ARV], we can obtain an analogy on compact non abelian groups.

§2. Notation.

Let U be a compact semisimple Lie group and $T\subset U$ a maximal torus of U. Let $\mathfrak{u}$ and $\mathfrak{t}$ denote the Lie algebras of U and T respectively, $\mathfrak{g}_C$ and $\mathfrak{t}_C$ the complexifications. The Haar measures du and dt are normalized by $\int_U du=\int_T dt=1$. Let $U^\wedge$ denote the set of all equivalence classes of

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