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# A Theorem of Pitman Type for Simple Random Walks on $Z^d$

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## Introduction.

Pitman's theorem ([2]) for a one-dimensional Brownian motion $B(t)$ states that $B(t)-2M(t)$ is a Bessel process of index 3, where $B(0)=0$ is assumed and $M(t)$ denotes the minimum of $B(s)$, $0\leqq s\leqq t$. This theorem can be obtained, after a scaling limit, from a similar theorem for a coin-tossing random walk on $\boldsymbol{Z}$ which is easy to prove and may still be called Pitman's theorem. An extension of Pitman's theorem to higher dimensional random walks is the following: given a simple random walk $\boldsymbol{S}_n$ on the $d$-dimensional lattice $\boldsymbol{Z}^d$ starting at 0, let $S_n^{(i)}$ be the $i$-th coordinate of $\boldsymbol{S}_n$ and denote by $M_n^{(i)}$ the minimum of $S_k^{(i)}$, $0\leqq k\leqq n$. Then the process

$$(1)\qquad \boldsymbol{S}_n-2\boldsymbol{M}_n=(S_n^{(1)}-2M_n^{(1)}, S_n^{(2)}-2M_n^{(2)}, \cdots, S_n^{(d)}-2M_n^{(d)})$$

ought to be a Markov chain. Unlike the corresponding statement for a higher dimensional Brownian motion, the above statement for $d\geqq 2$ is not an immediate consequence of the one for $d=1$ since the coordinate processes of $\boldsymbol{S}_n$ are not independent (in the case $d\geqq 2$). The purpose of this paper is to prove that $\boldsymbol{S}_n-2\boldsymbol{M}_n$ is a Markov chain on the $d$-dimensional (sub-)lattice $\boldsymbol{Z}_+^d$ of points with nonnegative integral coordinates (Theorem 1). Although a straightforward method used in the case $d=1$ (see §2) may also be applied to the case $d\geqq 2$, the argument will be quite messy. In this paper we employ another method which is based on the following simple observation: the coordinate processes of a simple random walk on $\boldsymbol{Z}^d$ ($d\geqq 2$) with *continuous* time are independent although this is not true for the case of discrete time.

## §1. Statement of the result.

Given an integer $d\geqq 2$, we write $\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_d$ for the $d$-dimensional

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