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On Certain Homogeneous Diophantine Equations of Degree $n(n-1)$

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1. In [3] Hilbert treated the Diophantine equation $D=D(x_0, x_1, \cdots, x_n)=\pm 1$, where

$$D=x_0^{2n-2}\prod(t_i-t_k)^2 \qquad (i=1, 2, \cdots, n;\ k=i+1, i+2, \cdots, n)$$

is the discriminant of

$$x_0t^n+x_1t^{n-1}+\cdots+x_n=0\ ,$$

with undetermined coefficients, and roots $t_1, t_2, \cdots, t_n$. He showed that, if $n>3$, the equation $D=\pm 1$ has no integer solutions. The proof is based on the theorem that the discriminant of an algebraic number field of degree $n>1$ is distinct from ± 1. Is his method applicable to other Diophantine equations?

In the present paper we discuss the homogeneous equation

$$a^s(n-1)^{n-1}x^{n(n-1)}+n^ny^{n(n-1)}=Az^{n(n-1)}\ , \tag{1.1}$$

where a, s, n, A are rational integers satisfying the following conditions:

(1) a is square-free, $|a|\neq 1$;

(2) $s\geqq 1$, $n\geqq 3$, $s<2(n-1)$, $A\neq 0$;

(3) $(n, asA)=((n-1)a, A)=1$.

The equation (1.1) may have non-trivial integer solutions; for example, if $A=a^s(n-1)^{n-1}+n^n$, then $x=y=z=1$ is a solution of (1.1). However, if A satisfies a certain condition, (1.1) has no integer solutions except $x=y=z=0$ (Theorem 1). The proof depends on a result of Komatsu [4] and Minkowski's inequality on the discriminant of an algebraic number field.

2. For simplicity, we shall use the following notation: For a prime

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