Incompressibility of Measured Laminations in 3-Manifolds

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Introduction. In this paper we study codimension-1 measured laminations in 3-manifolds. A lamination is a foliation in a closed subset of a manifold. A codimension-1 lamination is said to be measured when it has a transverse invariant measure. The basic study of codimension-1 measured laminations was done by Morgan and Shalen in [2]. Our aim is a further study of measured laminations in a 3-manifold, especially about its incompressibility.

In 3-dimension there is close relationship between measured laminations and branched surfaces defined by Floyd and Oertel in [1]. In Morgan-Shalen [2], it was proved that each leaf of a measured lamination carried by an incompressible branched surface is incompressible. On the other hand, our main theorem in this paper states that for any transversely orientable measured lamination each of whose leaves is incompressible and whose support is not the whole of the manifold, there exists an incompressible branched surface carrying it. Note that as our definition of measured laminations is different from that of Oertel [5], it is not easy to see even that for a measured lamination, there exists a branched surface carrying it. Hence first we must construct a branched surface carrying the measured lamination using a handle decomposition of the manifold. This constitutes the first half part of this paper.

After completing this paper, it was informed that A. Hatcher proved that a lamination whose leaves are incompressible is carried by an incompressible branched surface. (This result is still unpublished.) But his definition of lamination is the same as that of Oertel and different from ours. D. Gabai and U. Oertel also proved the above result with a little different method in a part of their work.

Throughout this paper we work in C^∞-category. The symbol M always denotes a closed orientable irreducible 3-manifold. By a term