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The Γ-Antilocality of Stable Generators Whose Lévy Measures are Supported on a Cone

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§1. Introduction.

The antilocality (to all directions) of an operator A states that, if $f=Af=0$ in a domain U then f is identically zero on the whole space. Goodman-Segal [2] and Murata [12] proved the antilocality in $\boldsymbol{R}^n$ of the operator $(m^2I-\Delta)^\lambda$, where Δ is the Laplacian and λ is a non-integral real number. The interest of the former physicists lied in the non-relativistic approximation of the relativistic quantum theory. Mathematically, both of them used classical methods connected with the Hardy space in the proof. Later, in his study on the complex powers of elliptic operators with analytic coefficients, Liess [11] showed the property using the hyperfunctions in connection with the theory of pseudodifferential operators with analytic symbols.

The notion of antilocality has been extended to that of the one-sided antilocality by Ishikawa [4], [5], and he proved it for the sums of one-dimensional stable generators. Recently, in connection with the potential theory corresponding to the operators of Lévy-Khintchin type, Kanda [8] has treated a similar problem.

Here we propose an extended notion of a biased antilocality (called Γ-antilocality). This states that if $f=Af=0$ in a domain U, then the zero's of f propagates from U to infinity only in some directions. This implies that if Γ is a (open or closed) convex cone in $\boldsymbol{R}^2$ and if $Af_1=Af_2$ and $f_1=f_2$ in a domain U for suitable functions f_1, f_2, then $f_1=f_2$ in $U+\Gamma$. Such an operator appears as an infinitesimal generator of a semi-group $(T_t)_{t>0}$ corresponding to a Markov process of jump type. In fact, the generator A of two-dimensional stable process whose Lévy measure is supported on a closed convex proper cone Γ has Γ-antilocality (Theorem

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