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Exponentially Bounded C-Semigroups and Integrated Semigroups

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Introduction.

Let X be a Banach space. We denote by $B(X)$ the set of all bounded linear operators from X into itself.

Let C be an injective operator in $B(X)$. We do not assume that the range $R(C)$ is dense in X. A family $\{S(t): t\geqq 0\}$ in $B(X)$ is called an *exponentially bounded C-semigroup on X*, if

(0.1) $$S(t+s)C=S(t)S(s) \quad \text{for } t, s\geqq 0 \text{ and } S(0)=C\,,$$

(0.2) $$S(\cdot)x: [0, \infty)\to X \text{ is continuous for } x\in X\,,$$

(0.3) there are $M\geqq 0$ and $a\in \boldsymbol{R}\equiv(-\infty, \infty)$ such that
$$\|S(t)\|\leqq Me^{at} \qquad \text{for} \quad t\geqq 0\,.$$

Let us define $L_\lambda\in B(X)$ for $\lambda>a$ by

$$L_\lambda x=\int_0^\infty e^{-\lambda t}S(t)xdt \qquad \text{for} \quad x\in X\,.$$

Similarly as in the case of $\overline{R(C)}=X$ (see [4]), we see that L_λ is injective for $\lambda>a$ and the closed linear operator Z defined by

(0.4) $$\begin{cases} D(Z)=\{x\in X: Cx\in R(L_\lambda)\} \\ Zx=(\lambda-L_\lambda^{-1}C)x \qquad \text{for} \quad x\in D(Z) \end{cases}$$

is independent of $\lambda>a$. The operator Z will be called the *generator* of $\{S(t): t\geqq 0\}$.

Recently, Davies and Pang [4] introduced the notion of an exponentially bounded C-semigroup under the assumption that $R(C)$ is dense in X and gave a characterization of the generator of an exponentially bounded C-semigroup. (See [3] also.) Later, the authors [6, 9, 11] gave a characterization of the complete infinitesimal generator of an exponentially

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