Exponentially Bounded $C$-Semigroups and Integrated Semigroups

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Introduction.

Let $X$ be a Banach space. We denote by $B(X)$ the set of all bounded linear operators from $X$ into itself.

Let $C$ be an injective operator in $B(X)$. We do not assume that the range $R(C)$ is dense in $X$. A family $\{S(t): t \geq 0\}$ in $B(X)$ is called an exponentially bounded $C$-semigroup on $X$, if

$$(0.1) \quad S(t+s)C = S(t)S(s) \quad \text{for } t, s \geq 0 \text{ and } S(0) = C,$$

$$(0.2) \quad S(\cdot)x: [0, \infty) \rightarrow X \text{ is continuous for } x \in X,$$

$$(0.3) \quad \text{there are } M \geq 0 \text{ and } a \in \mathbb{R} \equiv (-\infty, \infty) \text{ such that } \|S(t)\| \leq Me^{at} \quad \text{for } t \geq 0.$$

Let us define $L_\lambda \in B(X)$ for $\lambda > a$ by

$$L_\lambda x = \int_0^\infty e^{-\lambda t}S(t)x dt \quad \text{for } x \in X.$$

Similarly as in the case of $\overline{R(C)}=X$ (see [4]), we see that $L_\lambda$ is injective for $\lambda > a$ and the closed linear operator $Z$ defined by

$$\left\{ \begin{array}{ll}
D(Z) = \{x \in X: Cx \in R(L_\lambda)\} \\
Zx = (\lambda - L_\lambda^{-1}C)x \quad \text{for } x \in D(Z)
\end{array} \right.$$

is independent of $\lambda > a$. The operator $Z$ will be called the generator of $\{S(t): t \geq 0\}$.

Recently, Davies and Pang [4] introduced the notion of an exponentially bounded $C$-semigroup under the assumption that $R(C)$ is dense in $X$ and gave a characterization of the generator of an exponentially bounded $C$-semigroup. (See [3] also.) Later, the authors [6, 9, 11] gave a characterization of the complete infinitesimal generator of an exponentially

Received March 28, 1988