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Certain Random Motion of a Ball Colliding with Infinite Particles of Jump Type

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§0. Introduction.

In this paper we consider a system consisting of a hard ball with radius r and of infinitely many point particles moving in $\boldsymbol{R}^d$ according to the following rules:

(i) Let $x(t)$ be the center, the position, of the hard ball at time t. Then, there are no particles in $B_r(x(0))$ at time 0, where $B_r(x)$ denotes the r-neighborhood of x.

(ii) The ball or a particle at x waits an exponential holding time with mean one which is independent of the motion of the other particles. It jumps to the position y where y is distributed according to $p_x(dy)=p(|x-y|)dy$ independently of the holding time and the motion of the other particles, except that the jump is suppressed, if it causes a collision, that is, if there comes to lie a particle within the region occupied by the hard ball.

To give a precise description of the model we denote the position of infinite particles at time t by $\{y^i(t)\}_{i=1}^{\infty}$. We construct a Markov process describing an infinite particle system $\eta_t=\{z^i(t)\}_{i=1}^{\infty}$, where $z^i(t)=y^i(t)-x(t)$, which describes the entire configuration of particles seen from $x(t)$. We construct $x(t)$ as a functional of η_t. Let ν_0 be a Poisson distribution on $\boldsymbol{R}^d\backslash B_r(0)$ with intensity measure dx. Then, ν_0 is a stationary measure for η_t. The ergodicity of the stationary process is easily obtained. The main result of this paper is Theorem 2.1 which states that $\varepsilon x(t/\varepsilon^2)\to\sigma B(t)$ as $\varepsilon\to 0$, in the sense of distribution in $\boldsymbol{D}[0,\infty)$, where $B(t)$ is a d-dimensional Brownian motion and σ is a positive constant. We employ a method of Kipnis and Varadhan [2].

In §1 we construct a Markov process η_t and then $x(t)$ as a process driven by η_t. In §2 and §3 we prove the central limit theorem.

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