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The Rate of Convergence for Approximate Solutions of Stochastic Differential Equations

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§1. Introduction and results.

Let $(\Omega, \mathscr{F}, P)$ be a probability space and $B := \{B(t),\ t \geqq 0\} = \{(B^1(t), B^2(t), \cdots, B^r(t)),\ t \geqq 0\}$ an r-dimensional standard Brownian motion on it ($r \geqq 1$). We consider a stochastic differential equation (abbreviated by SDE) for a d-dimensional continuous process $X := \{X(t),\ 0 \leqq t \leqq 1\}$ ($d \geqq 1$):

$$dX(t) = \sigma(t, X(t))dB(t) + b(t, X(t))dt\ , \tag{1.1}$$

with $X(0) \equiv X_0$, where $\sigma(t, x) = \{\sigma_i^j(t, x),\ 1 \leqq i \leqq r,\ 1 \leqq j \leqq d\}$ is a Borel measurable function $(t, x) \in [0, 1] \times \boldsymbol{R}^d \to \boldsymbol{R}^d \otimes \boldsymbol{R}^r$ and $b(t, x) = \{b^j(t, x),\ 1 \leqq j \leqq d\}$ is a Borel measurable function $(t, x) \in [0, 1] \times \boldsymbol{R}^d \to \boldsymbol{R}^d$. Suppose that $\sigma(\cdot, \cdot)$ and $b(\cdot, \cdot)$ satisfy the following Lipschitz conditions: For any $x, y \in \boldsymbol{R}^r$ and $t, s \in [0, 1]$ there exists a positive constant L_1 independent of x, y, s and t such that

$$\|\sigma(t, x) - \sigma(s, y)\|^2 + |b(t, x) - b(s, y)|^2 \leqq L_1^2(|x - y|^2 + |t - s|^2)\ , \tag{1.2}$$

where

$$\|a\|^2 := \sum_{i=1}^{r} \sum_{j=1}^{d} |a_i^j|^2 \qquad \text{for} \quad a \in \boldsymbol{R}^d \otimes \boldsymbol{R}^r$$

and $|\cdot|$ denotes the Euclidean norm. Then there exists a unique solution of the SDE (1.1) (see, for example, Ikeda-Watanabe [8]). Approximate solutions for (1.1) were constructed by Maruyama [9], and its rate of convergence was studied by Gihman-Skorokhod [2] and Shimizu [17] (see also Greenside-Helfand [4], Janković [5], Janssen [6], Milshtein [10], Platen [11], [12], Rao-Borwanker-Ramkrishna [14], Rümelin [15], Wright [18]). In [2] and [17] on the rate of convergence, approximate solutions are

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