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Example of σ-Transition Matrices Defining the Horrocks-Mumford Bundle

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Introduction.

At the present state, the only examples of rank r indecomposable vector bundles on $\boldsymbol{P}^n$ with $2 \leqq r \leqq n-2$ are essentially the following: The rank 2 vector bundle on $\boldsymbol{P}^4$ constructed by Horrocks and Mumford [2]; the rank 2 on $\boldsymbol{P}^5$ in characteristic 2 by Tango [7]; the rank 3 on $\boldsymbol{P}^5$ in characteristic not equal to 2 by Horrocks [1]. About these bundles, especially about the Horrocks-Mumford bundle, many interesting facts have been discovered.

We here propose to focus our attention on systems of frames of these bundles and transition matrices with respect to them.

Recently, Tango [8] provided a theory of "σ-transition matrices", where his σ-transition matrices are transition matrices, defining a vector bundle on a projective space $\boldsymbol{P}^n$, over the standard covering of $\boldsymbol{P}^n$ carrying natural symmetry. Moreover, he showed that, for the Tango bundle and the Horrocks bundle above, there exist σ-transition matrices defining these bundles, respectively, and he actually computed these matrices. We note that arbitrary bundles on $\boldsymbol{P}^n$ are not necessarily defined by σ-transition matrices.

The purpose of this article is to find out rational sections of the Horrocks-Mumford bundle which give a system of frames over the standard covering of $\boldsymbol{P}^4$ and have the natural symmetry. Moreover, we shall write down the rational sections explicitly. It turns out that our frames are connected to each other by σ-transition matrices, and the Horrocks-Mumford bundle is also defined by σ-transition matrices.

Sasakura [5, 6] gave a theory of "configuration of divisors and reflexive sheaves". Using this theory, from a quite simple data consisting of transition matrices and divisors, he succeeded in reconstructing the Horrocks-Mumford bundle. In order to do this, he also found rational sections of the bundle, and computed transition matrices of the bundle

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