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Graded Buchsbaum Algebras and Segre Products

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§ 0. Introduction.

This paper is devoted to the study of Buchsbaum property in terms of certain spectral sequences and the application of this method to the investigation of Buchsbaum conditions for the Segre product of graded modules. In their paper [4], Goto and Watanabe showed the homogeneous version of Künneth's formula. This gives Cohen-Macaulay condition of the Segre product of graded modules. On the other hand, some sufficient conditions that the Segre product of graded modules have Buchsbaum property has been obtained by Stückrad and Vogel [15], and Schenzel [10]. The difficulty of seeking better sufficient conditions is caused by the difference between Buchsbaum property and quasi-Buchsbaum property. In other words, Buchsbaum property is not completely described in terms of local cohomology group without derived category. Our method is to describe Buchsbaum property in terms of spectral sequence. This approach leads to the three theorems in the end of this section.

In Section 1, we introduce the notion of r-Buchsbaum modules. In (1.8) and (1.11), we give a criterion of r-Buchsbaum property from the viewpoint of spectral sequence. As their corollaries, we have some inequalities concerning extension groups.

Section 2 is the homogeneous version of Section 1. The results of this section are applied in the later sections. Theorems 2.2 and 2.3 are the main theorems of this section. In (2.6) these give another proof of Stückrad and Vogel [16, Proposition 3.10]. Also, the theorems give how to calculate the dimension of some extension groups, for example, in (2.9).

Section 3 is, in some sense, an introduction to Section 4. In this section, we investigate some examples. In fact, the study of Example 3.4 gives the motivation to obtain our main theorems.

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