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On Asymptotic Stability for the Yang-Mills Gradient Flow

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Dedicated to Professor Hiroshi Fujita on his sixtieth birthday

§1. Introduction and statement of results.

The purpose of this paper is to study the asymptotic stability in $W^{m,r}$-sense for the Yang-Mills gradient flow around stable Yang-Mills connections.

We first concern with a closed connected Riemannian n-manifold (M, h) and consider a G-vector bundle $E=P\times_\rho \boldsymbol{R}^N$ associated with a G-principal bundle P over M. Here, G is a compact connected Lie group and ρ is a faithful orthogonal representation $\rho: G\to O_N$ of G.

On the space C_E of connections on E preserving the inner product of E, we consider the *Yang-Mills functional* (Y-M functional)

$$\mathrm{YM}(\nabla)=\frac{1}{2}\int_M |R^\nabla|^2 d_h x . \tag{1.1}$$

Here R^∇ and $d_h x$ denote the curvature tensor of connection ∇ and the Riemannian measure on (M, h), respectively and $|\ |$ is the norm determined by the inner product on E.

A critical point of the above functional (1.1) is called a *Yang-Mills connection* (a Y-M connection) and the corresponding curvature field is called the *Yang-Mills field* (the Y-M field), respectively. A Y-M connection is said to be *stable* if it minimizes (1.1) locally. Moreover, a Y-M connection ∇ is said to be *strictly stable* if the second variation of Y-M functional at ∇ is *strictly positive on a transversal orbit of the gauge group action* on C_E (see Definition 2.1). These notions are referred to Bourguignon-Lawson [3]. Typical examples of the stable Y-M connections are well-known self-dual connections on 4-sphere S^4. Moreover,

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