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3-Dimensional Fano Varieties with Canonical Singularities

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§0. Introduction.

In this article by a variety we mean an irreducible reduced projective variety over the field of complex numbers.

Let X be a 3-dimensional Fano variety with canonical singularities and H be a Cartier divisor satisfying $-K_X \sim r(X)H$ for the index $r(X)$ of X. The purpose of this article is to study the rational map $\Phi_{|H|}$ and singularities of a general member of $|H|$. In particular in the case of $r(X)=2$, which is the most essential, we find $\Phi_{|H|}$ to be as follows.
(1) When $d=H^3 \geqq 3$, a closed immersion into $\boldsymbol{P}^{d+1}$.
(2) When $d=2$, a double covering over $\boldsymbol{P}^3$.
And
(3) when $d=1$, a rational map that is defined except exactly one point and the closure of whose general fiber is a smooth elliptic curve.

And furthermore a general member S of $|H|$ has rational double points at $S \cap \mathrm{Sing}(X)$.

(0.1) DEFINITION. A variety V is called a *Fano variety* whenever the following conditions are satisfied.
(1) V is normal.
(2) V is Gorenstein, i.e. K_V is a Cartier divisor.
(3) The anticanonical divisor $-K_V$ is ample.

(0.2) DEFINITION. For an n-dimensional Fano variety V, we define the *index* of V to be $\max\{m \in \boldsymbol{Z} \mid \exists$ a Cartier divisor H such that $-K_V \sim mH\}$.

When V is an n-dimensional Fano variety, we use the following notation.

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