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An Example of a Normal Isolated Singularity with Constant Plurigenera δ_m Greater than 1

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Introduction. The plurigenera $\delta_m(X, x)$ of normal isolated singularities (X, x) were defined by Watanabe [4], as analogies of plurigenera P_m of complex manifolds. Thus δ_m have the properties similar to P_m. For instance, if P_m are bounded, then P_m are not greater than 1. The plurigenera of two-dimensional normal isolated singularities behave in the same way [1, Corollary 3.2]. However, higher dimensional normal isolated singularities may have the plurigenera δ_m greater than 1, although δ_m are bounded. The purpose of this paper is to give an example of such a normal isolated singularity.

Let $f: (\tilde{X}, E) \to (X, x)$ be a good resolution of an isolated singularity (X, x). Namely, each irreducible component E_i of the exceptional set $E = E_1 + E_2 + \cdots + E_s$ is a non-singular divisor on $\tilde{X}$ and E has only normal crossings as the singularities. We denote by C_i the divisor $\sum_{j \neq i} D_{ij}$ $(= E_i \cdot (E - E_i))$ on E_i, where D_{ij} is the intersection $E_i \cdot E_j$ of E_i and E_j.

DEFINITION [4, 5].

$$\delta_m(X, x) = \dim\{H^0(X \setminus \{x\}, \mathcal{O}_X(mK_X))/H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}(mK_{\tilde{X}} + (m-1)E))\} .$$

Here we note that the above definition does not depend on the choice of resolutions $(\tilde{X}, E) \to (X, x)$ by [2, Theorem 2.1].

THEOREM. *$\delta_m = s$ for each positive integer m, if*

$$\dim H^0(E_i, \mathcal{O}(mK_{E_i} + (k-m)[E_i]_{|E_i} + kC_i)) = \begin{cases} 0 & \text{for} \quad k > m > 0 \\ 1 & \text{for} \quad k = m > 0 , \end{cases}$$

for each E_i and if

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