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On the Fractal Curves Induced from the Complex Radix Expansion

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§0. Introduction.

Let α be a quadratic integer in a complex quadratic field $\boldsymbol{Z}(\sqrt{m}i)$ and $N\ (=N(\alpha))$ be the norm of α. Let $\mathscr{D}$ be a set of quadratic integers in $\boldsymbol{Z}(\sqrt{m}i)$ whose cardinality is equal to the norm of α, and denote it by

$$\mathscr{D}=\{r_0, r_1, \cdots, r_{N-1}\}\ , \qquad r_i \in \boldsymbol{Z}(\sqrt{m}i)\ .$$

A pair $(\alpha, \mathscr{D})$ is called a *number system* on $\boldsymbol{Z}(\sqrt{m}i)$ if every quadratic integer β in $\boldsymbol{Z}(\sqrt{m}i)$ is uniquely represented in the form

$$\beta=r_0+r_1\alpha+\cdots+r_j\alpha^j\ , \qquad r_i \in \mathscr{D} \quad (0\leqq i\leqq j) \tag{0.1}$$

and we say that β is expanded with *base* α and *digits* r_i $(0\leqq i\leqq j)$ if it is so represented. Most primitive example of the number system found in [9] and [10] is as follows: take $\alpha=i-1$ and $\mathscr{D}=\{0, 1\}$, then

1) $(\alpha, \mathscr{D})$ is a number system on Gaussian field $\boldsymbol{Z}(i)$, and
2) the Hausdorff dimension of the boundary of the set

$$X_{i-1}=\left\{\sum_{k=1}^{\infty} a_k(i-1)^{-k}\ \middle|\ a_k \in \mathscr{D}\right\}$$

is equal to

$$\frac{2\log\lambda}{\log 2}\fallingdotseq 1.5236$$

where λ is the positive root of $\lambda^3-\lambda^2-2=0$. This fact is extended as follows:

THEOREM (Katai-Szabo [8] and Gilbert [7]). *Let α be an integer in $\boldsymbol{Z}(i)$ and take $\mathscr{D}=\{0, 1, 2, \cdots, N-1\}$, then*

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