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A Transform on Classical Bounded Symmetric Domains Associated with a Holomorphic Discrete Series

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§1. Introduction.

In order to explain the aim of this paper we shall look at an example by taking the Poincaré model of the hyperbolic plane D and then consider its generalization.

1.1. Let D be the open unit disk $|z|<1$ in C with the usual manifold structure but given the Riemannian structure

$$ds^2=(1-x^2-y^2)^{-2}(dx^2+dy^2) \qquad (z=x+iy) . \tag{1.1}$$

Let $G=SU(1, 1)$ be the group of all C-linear transformations of C^2 preserving $|z_1|^2-|z_2|^2$ and of determinant one. Then each element g of G acts transitively on D as an analytic automorphism of D under

$$z\to z\cdot g=(\bar{\alpha}z+\beta)/(\bar{\beta}z+\alpha) \tag{1.2}$$

and $K=SO(2)$ is the subgroup of G fixing 0 in D, so we have the identification: $D=SO(2)\backslash SU(1, 1)$. If f is a complex valued function on D, its Fourier transform $f^\wedge$ on $C\times\partial D$, ∂D the boundary of D, is defined as follows:

$$f^\wedge(\lambda, b)=\int_D f(z)e^{(i\lambda+1)\langle z,b\rangle}dz \qquad (\lambda\in C,\ b\in\partial D) \tag{1.3}$$

for which this integral exists. Here $\langle z, b\rangle$ is the number given by the relation

$$e^{2\langle z,b\rangle}=(1-|z|^2)/|z-b|^2 \qquad (\lambda\in C,\ b\in\partial D) . \tag{1.4}$$

Then the characterization of $L^2(D)^\wedge$, the set of Fourier transforms of L^2 functions on D, is well-known as the Plancherel theorem on D (cf. [He],

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