On an Isomorphism between Specht Module and Left Cell of $\mathfrak{S}_n$

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Dedicated to Professor Nagayoshi Iwahori on his 60th birthday

Introduction.

The irreducible representations of the symmetric group $\mathfrak{S}_n$ are classically known and they are parametrized by the partitions of n. One of their realizations is so called Specht module which is defined over Z and has a natural base (Young's natural base). On the other hand, Kazhdan and Lusztig [3] constructed another realization, the W-graph representation which is obtained from a left cell of $\mathfrak{S}_n$ and it also has a natural Z-base (the vertices of the W-graph). We give in this paper an explicit isomorphism between the above two modules and show that the base change matrix is uni-triangular for some ordering of the base. Using this isomorphism, if a partition λ satisfies certain condition, we can construct the W-graph of the left cell of $\mathfrak{S}_n$ (without some edges which connect vertices having the same I-set) corresponding to λ not using the Kazhdan-Lusztig polynomials $P_{y,w}$ but using the relations in Specht module (the Garnir relations) inductively.

§1. The λ-diagram.

1.1. Notations. Let $P(n)$ be the set of partitions $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_r)$, where $\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_r > 0$ and $\sum_{i=1}^{r} \lambda_i = n$. For a partition λ, the Young frame of λ is the arrangement of n squares; the first row λ_1, the second row λ_2, $\cdots$, the last row λ_r parts, and line up to the left. Young tableau of shape λ has the frame λ and each square is numbered from 1 to n. A Young tableau of shape λ is called standard if its numbering is increasing from left to right in each row and from top to bottom in