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The Pseudo Orbit Tracing Property of First Return Maps

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Dedicated to Professor Kenichi Shiraiwa on his 60th birthday

§1. Introduction.

Every real flow without fixed points on a compact metric space induces a first return map on the union of sets in a certain family of local cross-sections, which was first introduced by H. Whitney [9] and after that improved by R. Bowen and P. Walters [2]. Our purpose is to investigate relationships between a real flow and its first return map with respect to the pseudo orbit tracing property.

H. B. Keynes and M. Sears [6] characterized already expansivity of a real flow by making use of a family of local cross-sections and a bijective first return map.

We denote by $(X, \boldsymbol{R})$ a real flow (abbrev. flow) without fixed points on a compact metric space X. Let d denote a metric for X and the action of $t \in \boldsymbol{R}$ on $x \in X$ is written xt. We write

$$SI=\{xt\ ;\ t \in I \text{ and } x \in S\}$$

for an interval I and $S \subset X$, and

$$\varepsilon_0=\inf\{t>0\ ;\ xt=x \text{ for some } x \in X\}\ .$$

Then ε_0 is a positive number since the flow $(X, \boldsymbol{R})$ has no fixed points and X is compact.

For positive numbers δ and α, a pair of doubly infinite sequences $(\{x_i\}_{i=-\infty}^{\infty}, \{t_i\}_{i=-\infty}^{\infty})$ is a *(δ, α)-chain* for $(X, \boldsymbol{R})$ if $t_i \geqq \alpha$ and $d(x_i t_i, x_{i+1})<\delta$ for all $i \in \boldsymbol{Z}$, and a pair of infinite sequences $(\{x_i\}_{i=0}^{\infty}, \{t_i\}_{i=0}^{\infty})$ is a *half (δ, α)-chain* for $(X, \boldsymbol{R})$ if $t_i \geqq \alpha$ and $d(x_i t_i, x_{i+1})<\delta$ for $i \geqq 0$. A (δ, α)-

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