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On the Action of Hecke Rings on Homology Groups of Smooth Compactifications of Siegel Modular Varieties and Siegel Cusp Forms

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Introduction and notations.

Let $g \geqq 1$ and $N \geqq 3$ be rational integers. We use the same notations as in Hatada [9]. Recall

1_g = the $g \times g$ unit integral matrix; $J_g = \begin{bmatrix} 0 & -1_g \\ 1_g & 0 \end{bmatrix}$;

$\Gamma = \Gamma_g(N)$ = the principal congruence subgroup of level N of $\mathrm{Sp}(g, \boldsymbol{Z})$ ($\subset \mathrm{GL}(2g, \boldsymbol{Z})$);

$\mathfrak{H}_g$ = the Siegel upper half plane of degree g;

$\Gamma \backslash \mathfrak{H}_g$ denotes the usual complex analytic quotient space;

$\mathrm{GSp}^+(g, \boldsymbol{R}) = \{\gamma \in \mathrm{GL}(2g, \boldsymbol{R}) \mid {}^t\gamma J_g \gamma J_g^{-1}$ is a scalar matrix whose eigenvalue is positive.$\}$;

$r(\alpha)$ = the eigenvalue of ${}^t\alpha J_g \alpha J_g^{-1}$ for $\alpha \in \mathrm{GSp}^+(g, \boldsymbol{R})$;

$\mathrm{GSp}^+(g, \boldsymbol{Z}) = \{\gamma \in \mathrm{GSp}^+(g, \boldsymbol{R}) \mid \gamma$ is an integral matrix.$\}$;

$\mathrm{GSp}^+(g, \boldsymbol{Q}) = \mathrm{GSp}^+(g, \boldsymbol{R}) \cap \mathrm{GL}(2g, \boldsymbol{Q})$;

$HR(\Gamma, \mathrm{GSp}^+(g, \boldsymbol{Z}))$ = the Hecke ring with respect to the group Γ and the monoid $\mathrm{GSp}^+(g, \boldsymbol{Z})$, cf. Hatada [8] and [9].

We consider the toroidal compactification of $\Gamma \backslash \mathfrak{H}_g$. We fix a regular and projective $\mathrm{Sp}(g, \boldsymbol{Z})$-admissible family of polyhedral cone decompositions: $\Sigma = \{\Sigma_\alpha\}_{F_\alpha:\ \text{rational components}}$ once for all. For example here we take a suitable refinement of the second Voronoi decomposition (cf. Namikawa [13], [14]). We write $(\Gamma \backslash \mathfrak{H}_g)^\sim$ for the projective smooth toroidal compactification of $\Gamma \backslash \mathfrak{H}_g$ with respect to this Σ. Write $M = (\Gamma \backslash \mathfrak{H}_g)^\sim$ for simplicity in this paper. For $\Gamma = \Gamma_g(N)$, define

$$\Gamma' = \{\xi \in \mathrm{Sp}(g, \boldsymbol{Z}) \mid \xi \ (\mathrm{mod}\ N) \text{ is a } 2g \times 2g \text{ diagonal matrix with coefficients in } \boldsymbol{Z}/N\boldsymbol{Z}.\},$$

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