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Three-Variable Conway Potential Function of Links

Dedicated to Professor Hiroshi Toda on his sixtieth birthday

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J. H. Conway introduced the potential function of a link in [3], and its invariance was verified by R. Hartley in [6]. Therefore we are not interested in its detailed definition in this note. We will try to give a recursive calculation of the potential functions of multi-variables. In fact we will succeed in giving it for the case of three variables as in Main Theorem. In other words, we will get a machine which makes any link into several links by a finite sequence of replacements appearing in Conway's three Identities for the case of three variables.

When we look back upon the past, we become aware that the existence of machines, which make any link into trivial knots by a finite sequence of replacements appearing in Conway's First Identity, has recently produced new polynomial invariants of links: the Jones polynomial and the skein polynomial [2, 4, 7, 8, 12, 14]. If we will get a machine for the case of multi-variables, it is possible to get a new (component-wise) link invariant.

For an ordered and oriented link in S^3, $L=K_1\cup\cdots\cup K_\mu$, we suppose that every component K_i is labeled by $t_{j(i)}$. The potential function $\nabla_L=\nabla_L(t_1,\cdots)$ has the following characterization [3], [6].

(I) (First Identity) For three links L_+, L_- and L_0 which differ only in one place as shown in Fig. 1, the potential function satisfies

$$\nabla_{L_+}=\nabla_{L_-}+(t_i-t_i^{-1})\nabla_{L_0}\ .$$

(II) (Second Identity) For three links L_{++}, L_{--} and L_{00} which differ only in one place as shown in Fig. 2 (a) or alternatively (b), the potential function satisfies

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