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Construction of Vector Bundles and Reflexive Sheaves

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§0. Introduction.

Let X be a smooth algebraic variety defined over a (not necessarily algebraically closed) field k. Let E be a vector bundle on X of rank $r-1$ $(r\geqq 2)$. Given a vector bundle F of rank r on X and an injection $\sigma: E\to F$, we can consider the closed subscheme $D(\sigma)=\{x\in X \mid \operatorname{rank}\sigma(x)< r-1\}$ of X. In §1, we discuss the relation between vector bundles and these closed subschemes associated with them. Our result is summarized as follows:

THEOREM (1.7). *Fix a vector bundle E as above and a line bundle L on X, and set $M=\det E$. Let $\mathscr{F}$ be the set of pairs (F, σ_F), where F is a vector bundle on X of rank r with* $\det F=L$, *and $\sigma_F: E\to F$ is an injection with $D(\sigma_F)$ of pure codimension* 2. *Let $\mathscr{G}$ be the set of pairs (Y, τ_Y), where Y is a Cohen-Macaulay closed subscheme of X of pure codimension* 2, *and $\tau_Y: E^\vee\to\omega_Y(-K_X+M-L)$ is a surjection. Then there exists a map $f:\mathscr{F}\to\mathscr{G}$ which is surjective in case $h^2(E(M-L))=0$.* (See (1.5), (1.6) and (1.7) for the precise statements.)

This theorem includes a result of Vogelaar [V] as a special case in which the following conditions are satisfied:
(1) X is a projective variety over an algebraically closed field,
(2) $E=\mathscr{O}_X^{\oplus r-1}$,
(3) Y is a locally complete intersection.
So our result is a generalization of that of Vogelaar's. We note that the above theorem also provides a way for constructing vector bundles. As an application, in §2, we will construct an indecomposable vector bundle of rank 3 on $\boldsymbol{P}^3$ which can never be obtained by Vogelaar's method.

In §3, we describe a method for constructing reflexive sheaves from

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