TOKYO J. MATH.
VOL. 13, NO. 1, 1990

Remarks on Bayes Sufficiency

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Introduction.

It is well known that there are various definitions of "sufficiency". In addition to the usual definition of sufficiency represented by the existence of a common conditional probability, we have the notions of pairwise sufficiency, PSS (pairwise sufficiency with supports), test sufficiency and Bayes sufficiency. These notions coincide with one another in the dominated case. Ramamoorthi ([6]), and Roy and Ramamoorthi ([7]), discussed Bayes sufficiency in undominated cases, but most results are restricted to the countably generated subfield cases.

In this note we show, by examples, that PSS does not imply Bayes sufficiency in case of a continuous a priori distribution (cf. [5]), and the existence of the smallest PSS which is Bayes sufficient. This latter example shows the result by Kusama and Fujii ([4]) does not hold if we replace test sufficiency by Bayes sufficiency.

By a statistical experiment we mean a triplet $(\mathscr{X}, \mathscr{A}, \mathscr{P})$, where $\mathscr{P}=\{P_\theta; \theta \in \Theta\}$ is a family of probability measures on $(\mathscr{X}, \mathscr{A})$. Θ is referred to as the parameter space of the experiment. Let $\mathscr{C}$ be a sigma-field of subsets of Θ which includes the sigma-field generated by the family of mappings defined by $\theta \in \Theta \to P_\theta(A)$, $A \in \mathscr{A}$. For any a priori distribution λ on $(\Theta, \mathscr{C})$, let $\mathscr{P}*\lambda$ be a probability measure on $(\mathscr{X} \times \Theta, \mathscr{A} \times \mathscr{C})$ defined by

$$\mathscr{P}*\lambda(A \times C) = \int_C P_\theta(A) d\lambda(\theta) .$$

Let $\mathscr{B}$ be a sub-sigma-field of $\mathscr{A}$ and Λ be a family of a priori distributions on $(\Theta, \mathscr{C})$.

DEFINITION. $\mathscr{B}$ is called Bayes sufficient w.r.t. Λ if, for any $\lambda \in \Lambda$, $\mathscr{A} \times \Theta$ and $\mathscr{X} \times \mathscr{C}$ are conditionally independent given $\mathscr{B} \times \Theta$ w.r.t.

Received June 22, 1989
Revised October 1, 1989