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Examples on an Extension Problem of Holomorphic Maps and a Holomorphic 1-Dimensional Foliation

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§0. Introduction.

Let C^2 be the two dimensional complex vector space with a standard system of coordinates $z=(z_1, z_2)$. Put

$$B=\{z\in C^2 : |z|<1\} ,$$
$$\partial B(\varepsilon)=\{z\in C^2 : 1-\varepsilon<|z|<1\} ,$$
$$\Sigma_1=\{z\in C^2 : |z|=1\} , \quad \text{and}$$
$$\Sigma_2=\{z\in C^2 : |z|=1-\varepsilon\} ,$$

where ε is a constant such that $0<\varepsilon<1$, and

$$|z|^2=|z_1|^2+|z_2|^2 .$$

In this note, first *we shall construct compact complex 3-folds M which admit a holomorphic map*

$$f : \partial B(\varepsilon) \longrightarrow M$$

such that the inner boundary Σ_2 of $\partial B(\varepsilon)$ is a natural boundary of f. That is, for any point $x\in\Sigma_2$, we *cannot* find any neighborhood W of x in C^2 such that f can be extended to a holomorphic map of $W\cup\partial B(\varepsilon)$ into M. Secondly, we study a 1-dimensional holomorphic foliation on the associated projective bundle $P(TM)$ of the tangent bundle TM. We shall show that *in $P(TM)$ there are a subdomain W, $P(TM)-[W]\neq\emptyset$, and a thin subset S of $P(TM)-[W]$ such that every leaf in W is biholomorphic to P^1 and all compact leaves outside $[W]$ are contained in S*, where $[W]$ indicates the closure of W in $P(TM)$.

In §1, we shall construct our compact complex 3-fold M. In §2, we shall prove the non-extendibility of a certain holomorphic map into M (see also [2]). In §3, we study the holomorphic foliation on $P(TM)$.

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