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On the Convergence of Series of Fourier Coefficients of Vector Valued Functions

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§ 1. Introduction.

Let X be a Banach space with norm $\|\cdot\|$ and let $x(t)$ be an X valued function on $-\infty<t<\infty$ which is 2π periodic: $\|x(t+2\pi)-x(t)\|=0$ for every t, and integrable (in Bochner sense [2]) on $T\equiv(-\pi, \pi)$. One may define the Fourier coefficients of $x(t)$:

$$c_n=\frac{1}{2\pi}\int_{-\pi}^{\pi} x(t)e^{-int}dt\ , \qquad n=0,\ \pm 1, \cdots . \tag{1}$$

Kandil [6] has studied the unconditional convergence of $\sum_{n=-\infty}^{\infty} c_n$ in X and the convergence of $\sum_{n=-\infty}^{\infty} \|c_n\|$ when X is a Hilbert space, and he gave sufficient conditions for those sorts of convergence, which are analogous to the known criteria for the absolute convergence of ordinary Fourier series of complex valued functions.

The purpose of this paper is to generalize Kandil's results as well as the author's theorem on the convergence of $\sum \|c_n\|$ when X is the space of random variables [7], [8].

We introduce some notations. Let $L^p(T)$ be the class of complex valued functions $f(t)$ with $\int_T |f(t)|^p dt<\infty$, $p>0$, as usual. The class of $x(t)$ with $\|x(t)\|\in L^p(T)$ is denoted by $\boldsymbol{L}^p(T)$, $p\geqq 1$. We remark that such function $x(t)$ is integrable. Write

$$|||x(\cdot)|||_p=\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} \|x(t)\|^p dt\right]^{1/p}\ , \qquad p\geqq 1\ .$$

Letting

$$\Delta_h^{(r)}x(t)=\sum_{k=0}^{r} (-1)^{r-k}\binom{r}{k}x(t+kh)\ ,$$

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