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Interpolation between Some Banach Spaces in Generalized Harmonic Analysis

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Dedicated to Professor Sumiyuki Koizumi on his sixtieth birthday

Introduction.

In [14, 15], N. Wiener established the generalized harmonic analysis for the analysis of almost periodic functions and sample paths of the Brownian motions. The classes of functions he treated are

$$(0.1)\qquad W^2(\boldsymbol{R}^1)=\left\{f\in L^2_{\mathrm{loc}}(\boldsymbol{R}^1)\ :\ \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|f(x)|^2dx \text{ exists}\right\}$$

and its subclasses. The $\boldsymbol{R}^2$ case of the generalized harmonic analysis was investigated by K. Anzai, S. Koizumi and K. Matsuoka [1] and K. Matsuoka [10, 11], and also the $\boldsymbol{R}^n$ case by T. Kawata [7].

Unfortunately, the class $W^2(\boldsymbol{R}^1)$ is not closed under addition. Hence, the following two more conventional Banach spaces were considered:

$$(0.2)\qquad M^p(\boldsymbol{R}^1)=\left\{f\in L^p_{\mathrm{loc}}(\boldsymbol{R}^1)\ :\ \|f\|_{M^p(\boldsymbol{R}^1)}=\varlimsup_{T\to\infty}\left(\frac{1}{2T}\int_{-T}^{T}|f(x)|^pdx\right)^{1/p}<\infty\right\},$$

which is called the Marcinkiewicz space, and

$$(0.3)\qquad B^p(\boldsymbol{R}^1)=\left\{f\in L^p_{\mathrm{loc}}(\boldsymbol{R}^1)\ :\ \|f\|_{B^p(\boldsymbol{R}^1)}=\sup_{T\geq 1}\left(\frac{1}{2T}\int_{-T}^{T}|f(x)|^pdx\right)^{1/p}<\infty\right\},$$

where $1<p<\infty$. Recently, K. Lau [8, 9] investigated the multiplier theory on $M^p(\boldsymbol{R}^1)$. Also, Y. Chen and K. Lau [5] developed the harmonic analysis on $B^p(\boldsymbol{R}^1)$ and the related spaces (e.g., the Hardy-Littlewood maximal function, the Hardy spaces, John-Nirenberg's *BMO*, the Carleson measure, the atomic decomposition, and Fefferman-Stein's duality).

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