TOKYO J. MATH.
VOL. 13, NO. 1, 1990

Classifying 3-Dimensional Lens Spaces by Eta-Invariants

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Dedicated to Professor Akio Hattori on his sixtieth birthday

Let C^2 be the space of pairs (z_0, z_1) of complex numbers with the standard flat Kähler metric. Let p be a positive integer and q_0, q_1 be integers relatively prime to p. Put $z=\exp\frac{2\pi\sqrt{-1}}{p}$ and define an isometry g of C^2 by

$$g: (z_0, z_1) \longrightarrow (z^{q_0}z_0, z^{q_1}z_1) .$$

Then g generates a cyclic subgroup $G=\{g^k\}_{k=0,1,\cdots,p-1}$ of the unitary group $U(2)$ and the elements g^k act on the unit sphere

$$S^3=\{(z_0, z_1)\in C^2;\ z_0\bar{z}_0+z_1\bar{z}_1=1\}$$

without fixed point. The differentiable manifold S^3/G has a unique riemannian metric so that the covering projection $\varphi: S^3\to S^3/G$ gives a local isometry of S^3 onto S^3/G. This riemannian manifold S^3/G is called a lens space and is denoted by $L(p; q_0, q_1)$.

The following theorem on lens spaces is well known. (See Cohen [3].)

THEOREM. *The following assertions are equivalent*:

(1) $L(p; q_0, q_1)$ *is isometric to* $L(p; \bar{q}_0, \bar{q}_1)$.

(2) $L(p; q_0, q_1)$ *is diffeomorphic to* $L(p; \bar{q}_0, \bar{q}_1)$.

(3) $L(p; q_0, q_1)$ *is homeomorphic to* $L(p; \bar{q}_0, \bar{q}_1)$.

(4) *There are integers* l *and* $e_i\in\{-1, 1\}$ $(i=0, 1)$ *such that* (q_0, q_1) *is a permutation of* $(e_0l\bar{q}_0, e_1l\bar{q}_1)$.

Since g^k is also a generator of G, the lens space $L(p; kq_0, kq_1)$ is identical to $L(p; q_0, q_1)$. Hence, choosing a suitable generator for G, we

Received April 13, 1989
Revised May 26, 1989