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On the Equation $s(1^k+2^k+\cdots+x^k)+r=by^z$

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§1. Introduction.

We consider the equation

$$s(1^k+2^k+\cdots+x^k)+r=by^z \tag{1}$$

where b, s, r, and k are integer constants and investigate the conditions under which we can assert that the equation has only finitely many solutions in integers $x>0$, $y\geqq 2$, and $z\geqq 2$.

This was proved by K. Györy, R. Tijdeman and M. Voorhoeve [4] in the case $b\neq 0$, $k>0$, $s=1$, and r arbitrary, provided that $k\notin\{1, 3, 5\}$. They also stated the same condition when s is a certain squarefree odd integer.

B. Brindza [2] proved the assertion in the case when s is squarefree and $z\notin\{1, 2, 3, 4, 6\}$ or if s is odd and $k\notin\{1, 2, 3, 5\}$.

In this paper, we obtain new conditions on k, r, and s which allow us to show that (1) has only finitely many solutions in integers $x>0$, $|y|\geqq 2$, and $z\geqq 2$.

§2. Results.

For an integer $n\neq 0$ and a prime p, there exists an integer $m\geqq 0$ for which $p^m \| n$. Then we put $\nu_p(n)=m$ and define, for a nonzero rational number $\alpha=m/n$ with $m, n\in \boldsymbol{Z}$,

$$\nu_p(\alpha)=\nu_p(m)-\nu_p(n)$$

which depends only on α. Also we write $\operatorname{num}\alpha=m$ and $\operatorname{den}\alpha=n$ for a rational number $\alpha=m/n$ with $m, n\in\boldsymbol{Z}$, $n>0$, and $(m, n)=1$, where (m, n) denotes the greatest common divisor of m and n.

THEOREM. *For given integers $b\neq 0$, $r\neq 0$, $s\neq 0$, and $k>0$, the equation*

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