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Pitman Type Theorem for One-Dimensional Diffusion Processes

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Introduction.

For a regular diffusion X in $\boldsymbol{R}$ starting from 0 with generator $\mathscr{L}=d/dm\cdot d/ds$, $s(0)=0$, and for any fixed $r>0$, define

$$\tau_1=\inf\{t>0 : X(t)=r\} ,$$
$$\tau_2=\sup\{t>0 : X(t)=0,\ t<\tau_1\} .$$

Then Williams's theorem ([4]) states that $\{X(\tau_2+t) : 0\leqq t\leqq\tau_1-\tau_2\}$ is identical in distribution to $\{\tilde{X}(t) : 0\leqq t\leqq\tilde{\tau}\}$, where $\tilde{X}$ is a diffusion process starting from 0 with generator $\tilde{\mathscr{L}}f=(1/s)\mathscr{L}(sf)$ and $\tilde{\tau}=\inf\{t>0 : \tilde{X}(t)=r\}$. In the case where X is a one-dimensional Brownian motion B with $B(0)=0$, $\tilde{X}$ is a Bessel process with index 3 (the radial part of a three-dimensional Brownian motion) and Pitman [2] proved that

$$(1)\qquad \{\tilde{X}(t),\ t\geqq 0\}\stackrel{\mathrm{d}}{=}\{B(t)+2L(t),\ t\geqq 0\} ,\qquad L(t)=-\min_{0\leqq u\leqq t} B(u) ,$$

where "$\stackrel{\mathrm{d}}{=}$" means the equality in distribution.

In this paper we consider the case where X is the one-dimensional diffusion process defined by the stochastic differential equation (abbreviated : SDE)

$$(2)\qquad X(t)=\int_0^t\sigma(X(u))dB(u)+\int_0^t b(X(u))du ,$$

and will prove that $\tilde{X}$ admits a representation similar to (1) (see Theorem 1.1′). The assumptions for the coefficients σ and b are that they are Lipschitz continuous and $\sigma(x)>0$, $\forall x\in\boldsymbol{R}$. To state our result more pre-

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