TOKYO J. MATH.
VOL. 13, NO. 2, 1990

Hilbert Spaces of Analytic Functions and the Gegenbauer Polynomials

Shigeru WATANABE

Sophia University
(Communicated by N. Iwahori)

Introduction.

Let $\boldsymbol{F}$ be the Fock type Hilbert space of analytic functions $f(z)$ of n complex variables $z=(z_1, z_2, \cdots, z_n)\in \boldsymbol{C}^n$, with the scalar product

$$(f, g)=\pi^{-n}\int_{\boldsymbol{C}^n}\overline{f(z)}g(z)\exp(-|z_1|^2-\cdots-|z_n|^2)dz_1\cdots dz_n ,$$

with

$$dz_1\cdots dz_n=dx_1\cdots dx_n dy_1\cdots dy_n , \qquad z_j=x_j+iy_j ,$$

and let $\boldsymbol{H}$ be the usual Hilbert space $L^2(\boldsymbol{R}^n)$. Bargmann constructed in [1] a unitary mapping A from $\boldsymbol{H}$ to $\boldsymbol{F}$ given by an integral operator whose kernel is related in some definite sense to the Hermite polynomials. More precisely, $f=A\phi$ for $\phi\in\boldsymbol{H}$ is defined by

$$f(z)=\int_{\boldsymbol{R}^n}A(z, q)\phi(q)d^n q$$

with

$$A(z, q)=\pi^{-n/4}\prod_{j=1}^{n}\exp\left\{-\frac{1}{2}(z_j^2+q_j^2)+2^{1/2}z_j q_j\right\} .$$

The purpose of the present paper is to show that similar constructions are possible for some other classical orthogonal polynomials.

§1. The arguments for the Gegenbauer polynomials.

Let λ be a positive real number. The Gegenbauer polynomials C_m^λ, $m=0, 1, 2, \cdots$, are defined as the coefficients in the expansion

Received December 22, 1989