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On Peak Sets for Certain Function Spaces II

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Introduction.

This paper is the continuation of [15]. In [15], the authors showed that some theorems on function algebras can be generalized to the case of a wider class of function spaces containing the class of function algebras. This class is of function spaces having the condition (A) (see §1). In this paper, we introduce the conditions (B) and (C) which are weaker than (A). In §1, we discuss the conditions (B) and (C), and give some examples connected with them. In §2, we consider the class $\mathscr{B}$ of function spaces having (B) and the class $\mathscr{C}$ of function spaces having (C), and give characterizations to assert that $A=C(X)$ for $A\in\mathscr{B}$ or $A\in\mathscr{C}$. Especially, we establish generalizations of a theorem of Rudin [13] and a theorem of Hoffman and Wermer [11] (Theorems 2.1 and 2.5).

§1. Conditions for function spaces.

Throughout this paper, X will denote a compact Hausdorff space. A is said to be a *function space* (resp. *function algebra*) on X if A is a closed subspace (resp. subalgebra) in $C(X)$ containing constant functions and separating points in X, where $C(X)$ denotes the Banach algebra of complex-valued continuous functions on X with the supremum norm.

Let A be a function space on X. For a subset E in X, we denote

$$A(E)=\{f\in C(E): fg\in A|_E \text{ for any } g\in A|_E\}$$
$$A_R(E)=\{f\in C_R(E): fg\in A|_E \text{ for any } g\in A|_E\}\ ,$$

where $A|_E$ is the restriction of A to E and $C_R(E)$ is the set of all real-valued continuous functions on E.

Let E be a subset in X. Then we call E an *antisymmetric set* for A if any function in $A_R(E)$ is constant. We write $\mathscr{K}(A)$ the family

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