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Time Reversal of Random Walks in R^d

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Introduction.

The purpose of this paper is to give an extension, to a higher dimensional case, of the result [3] concerning time reversal of random walks.

Suppose we are given a pseudo-order $\lhd$ in $\boldsymbol{R}^d$ such that $x \lhd y$ implies $x+z \lhd y+z$ for any $z \in \boldsymbol{R}^d$. We write $x \blacktriangleleft y$ if $x \lhd y$ and $x \neq y$, and put $K=\{x \in \boldsymbol{R}^d : 0 \lhd x\}$. Then $x \lhd y$ if and only if $y-x \in K$. The set K contains 0 and satisfies

$$(1) \qquad x+y \in K \quad \text{if} \quad x, y \in K .$$

Throughout the paper we assume that the set K is infinite and Borel. Given a random walk $S_n=\sum_{k=1}^{n} X_k$ in $\boldsymbol{R}^d$, we define a random time τ by

$$(2) \qquad \tau=\min\{n \geqq 1 : S_n \blacktriangleleft S_k \text{ for } 0 \leqq \forall k \leqq n-1\} ,$$

and assume that $\tau<\infty$ a.s. One more assumption, which is technical and might probably be removed, is that the random walk is countably valued, namely, if Γ denotes the (countable) set of x such that $P\{X_k=x\}>0$ then

$$(3) \qquad P\{X_k \in \Gamma\}=1 .$$

Next we consider the time reversal

$$(4) \qquad (0,\ S_{\tau-1}-S_\tau,\ S_{\tau-2}-S_\tau, \cdots, S_1-S_\tau,\ -S_\tau)$$

and regard this as a (finite length) path-valued random variable. Taking independent copies w_k, $k \geqq 1$, of (4), we define a process $\{W_n,\ n \geqq 0\}$ by (1.1). Then our main result is that $\{W_n,\ n \geqq 0\}$ is a Markov chain with transition function $\hat{p}_\xi(x, y)$ given by (1.3). The result of [3] is a special

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