TOKYO J. MATH.
VOL. 13, NO. 2, 1990

Hyperbolic 3-Manifolds with Totally Geodesic Boundary Which Are Decomposed into Hyperbolic Truncated Tetrahedra

Michihiko FUJII

Tokyo Institute of Technology
(Communicated by K. Ogiue)

§0. Introduction.

In this paper we study compact oriented hyperbolic 3-manifolds each of which has a totally geodesic boundary. By a hyperbolic manifold, we mean a Riemannian manifold with constant sectional curvature -1. We construct such 3-manifolds by gluing the hexagonal faces of hyperbolic truncated tetrahedra by isometries, where a hyperbolic truncated tetrahedron is a polyhedron in the hyperbolic 3-space $\boldsymbol{H}^3$ bounded by four totally geodesic right-angled hexagons and four triangles (precise definition is given by Proposition 2.1). Such a construction was presented by W. P. Thurston in a lecture at the University of Warwick in July in 1984 (the author learned it from Professor Sadayoshi Kojima).

We deal with a detailed study of constructing hyperbolic 3-manifolds by gluing hyperbolic truncated tetrahedra in §1 and §2, and after these preparations, we will show the following result in §3.

THEOREM 3.1. *There are exactly eight mutually non-isometric compact oriented hyperbolic* 3-*manifolds with totally geodesic boundary such that they can be decomposed into two hyperbolic truncated tetrahedra and that their boundaries are closed surfaces of genus* 2.

We can easily obtain the presentations of fundamental groups of the above eight 3-manifolds. But it is quite difficult to distinguish them, even though we compute the first homology group of the n-fold covering of each of them by using the Reidemeister-Schreier method (for each $n \geqq 2$). So we shall distinguish the above eight 3-manifolds geometrically by using the shortest return path, where a return path is a geodesic arc which starts from and returns back to the boundary surface with the right angle.

Received October 26, 1989