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Surfaces with Constant Kaehler Angle All of Whose Geodesics Are Circles in a Complex Space Form

Sadahiro MAEDA and Seiichi UDAGAWA

Kumamoto Institute of Technology and Nihon University
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Dedicated to Professor Tadashi Nagano on his sixtieth birthday

§0. Introduction.

Let $f: M \to \tilde{M}$ be an isometric immersion of a connected complete Riemannian manifold M into a Riemannian manifold $\tilde{M}$. We call M *a circular geodesic* submanifold of $\tilde{M}$ provided that for every geodesic γ of M the curve $f \circ \gamma$ is a circle in $\tilde{M}$. The following problem is still open: Classify circular geodesic submanifolds M in a complex space form (for details, see [7]).

The purpose of this paper is to consider this problem in the case of $\dim M = 2$.

§1. Preliminaries.

A Riemannian manifold of constant curvature is called a *real space form*. Let M be an n-dimensional submanifold of $\tilde{M}^{n+p}$ with the metric g. We denote by ∇ and $\tilde{\nabla}$ the covariant differentiations on M and $\tilde{M}$, respectively. Then, the second fundamental form σ of the immersion is defined by $\sigma(X, Y) = \tilde{\nabla}_X Y - \nabla_X Y$, where X and Y are the vector fields tangent to M. We call $\mu = (1/n)(\text{trace } \sigma)$ the *mean curvature vector* of M in $\tilde{M}$. The *mean curvature* H of M in $\tilde{M}$ is the length of μ. If μ is identically zero, the submanifold is said to be *minimal*. The submanifold M is *totally umbilic* provided that $\sigma(X, Y) = g(X, Y)\mu$ for all vector fields X and Y on M. In particular, if σ vanishes identically, then M is said to be a *totally geodesic* submanifold of $\tilde{M}$. For a vector field ξ normal to M, we write $\tilde{\nabla}_X \xi = -A_\xi X + D_X \xi$, where $-A_\xi X$ (resp. $D_X \xi$) denotes the tangential (resp. the normal) component of $\tilde{\nabla}_X \xi$. We call D

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