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The Left Cells and Their W-Graphs of Weyl Group of Type F_4

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§1. Introduction.

Let (W, S) be a finite Coxeter system. In [5], Kazhdan and Lusztig constructed W-graphs. They are obtained from the left cells of W and give the representations of Hecke algebra $\mathscr{H}$ corresponding to W. In this paper, we assume (W, S) has type F_4. The Kazhdan-Lusztig polynomials $P_{y,w}$ for (W, S) of type F_4 are already calculated (see [13]). In Section 2, we recall the definition of $P_{y,w}$ and several relations on W. The main results of this paper appear in Section 3. Using some data of $P_{y,w}$, we determine the left cells and two-sided cells. The left and two-sided cells are explicitly constructed from certain easily described subsets of W (Theorem 3.1), and we describe the natural W-graph corresponding to each left cell (Theorem 3.2). After describing each W-graph, we discuss some relations between the Duflo involutions and the conjugate classes in W by examining each case (Proposition 3.6).

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§2. Preliminaries.

Let (W, S) be an arbitrary Coxeter system, with the Bruhat order "$\leqq$" and the length function $l: W \to N$. Let $\mathscr{A} = Z[q^{1/2}, q^{-1/2}]$ be the ring of Laurent polynomials in $q^{1/2}$ where $q^{1/2}$ is an indeterminate and let $\mathscr{H}$ be the Hecke algebra of (W, S) over $\mathscr{A}$ with standard basis $\{T_w \mid w \in W\}$. In [5], Kazhdan and Lusztig defined the special basis $\{C_w \mid w \in W\}$ for $\mathscr{H}$, given by

$$C_w = \sum_{y \leqq w} (-1)^{l(w)-l(y)} q^{l(w)/2-l(y)} P_{y,w}(q^{-1}) T_y \,,$$

where $P_{y,w} \in \boldsymbol{Z}[q]$ is a polynomial in q of degree $\leqq (l(w)-l(y)-1)/2$ for

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