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The Characters of a Maximal Parabolic Subgroup of $GL_n(\boldsymbol{F}_q)$

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§0. Introduction.

Let G_n be the general linear group $GL_n(\boldsymbol{F}_q)$, that is, the group of nonsingular matrices of degree n with coefficients in the finite field $\boldsymbol{F}_q$ and P_n be the maximal parabolic subgroup of G_n consisting of matrices $(g_{ij})\in G_n$ such that $g_{21}=g_{31}=\cdots=g_{n1}=0$.

In this paper we show an inductive method to calculate the irreducible characters of P_n and also determine the branching rules of irreducible characters for $G_n\to P_n$ and $P_n\to L_n$, where L_n is a Levi subgroup of P_n and hence L_n is isomorphic to $G_1\times G_{n-1}$. J. A. Green [2] showed how to calculate the irreducible characters of G_n and A. V. Zelevinsky [5] determined the branching rules for $G_n\to H_n$ and $H_n\to G_{n-1}$, where $H_n=\{(g_{ij})\in P_n \mid g_{11}=1\}$ is the group of affine transformations. Thus this paper can be viewed as an application of these two papers.

We use the following notation. We deal here only with complex characters and so for a finite group G, $\mathrm{Irr}\,G$ stands for the set of all irreducible complex characters of G and $\mathrm{ch}\,G$ is the ring of virtual complex characters of G. For a subgroup H of G and $\varphi\in\mathrm{ch}\,H$, $\mathrm{Ind}_H^G\varphi$ is the induced character of φ from H to G and for $\chi\in\mathrm{ch}\,G$, $\mathrm{Res}_H^G\chi$ is the restriction of χ to H. For χ_1, $\chi_2\in\mathrm{ch}\,G$,

$$(\chi_1, \chi_2)_G=|G|^{-1}\sum_{g\in G}\chi_1(g)\chi_2(g^{-1}) .$$

For $g\in G$, $Z_G(g)$ is the centralizer of g in G and for a fiinite set X, $|X|$ denotes its cardinality. Moreover we denote by $\boldsymbol{N}$ the set of natural numbers, so that $\boldsymbol{N}=\{0, 1, 2, \cdots\}$. Now let $V_n=\boldsymbol{F}_q{}^n$ be the n-dimensional vector space of column n-vectors with coefficients in $\boldsymbol{F}_q$ and $(e_1, e_2, \cdots, e_n)$ be the canonical basis of V_n. Then V_n is naturally a left G_n-module. For $f\in\mathrm{End}\,V_n$, $\mathrm{Im}\,f$ is the image of f and $\mathrm{Ker}\,f$ is the kernel of f. We also regard $\boldsymbol{F}_q$ to be the subset of $\mathrm{End}\,V_n$ identifying $a\in\boldsymbol{F}_q$ with $a\cdot 1_{V_n}$.

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