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# Generalization of Lucas' Theorem for Fermat's Quotient II

Nobuhiro TERAI

*Waseda University*
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## Introduction.

Let $p$ be an odd prime number and let $m$ be a positive integer prime to $p$. We define Fermat's quotient $q_p(m)$ by $q_p(m)=\frac{m^{p-1}-1}{p}$. Lucas ([2], [5]) proved that $q_p(2)$ is a square only for $p=3$ and 7. To generalize Lucas' theorem, we consider whether the equation

$$(*) \qquad q_p(m)=x^l$$

has solutions or not, where $l$ is a prime and $x$ is a positive integer.

In the previous paper [9], we considered the three cases of $(*)$:

$$(\mathrm{I}) \qquad q_p(m)=x^2 \qquad (p>3)$$

$$(\mathrm{II}) \qquad q_p(r)=x^r \qquad (r \text{ is an odd prime})$$

$$(\mathrm{III}) \qquad q_p(2)=x^l \qquad (l \text{ is an odd prime})$$

and we obtained the following three theorems:

THEOREM A. *If $m$ is odd, then the equation* (I) *has the only solution* $(p, m, x)=(5, 3, 4)$.

THEOREM B. *If the equation* (II) *has solutions, then $p$ and $r$ satisfy the congruences*

$$2^{r-1}\equiv 1 \pmod{r^2} \quad \textit{and} \quad p^{r-1}\equiv 1 \pmod{r^2}\,.$$

THEOREM C. *The equation* (III) *has the only solution* $p=3$.

In this paper, we treat more general cases of $(*)$. In §1, we discuss the equation $(*)$ when $m$ is even and $p>3$. Then it is proved that if Catalan's conjecture holds, namely, if the only solution in integers $m>1$,

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