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Prüfer Domain and Affine Scheme

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Introduction.

Valuation theory has an intimate relation with number theory and algebraic geometry. The following Theorem 0, for instance, shows a role of discrete valuation rings.

Let K be a field. We consider the following four conditions for a set W consisting of valuation rings with quotient field K:

(W-0) $W \neq \emptyset$.

(W-1) If $R \in W$, then R is a discrete valuation ring.

(W-2) For any $x \in K$, the set $\{R \in W \mid x \notin R\}$ is finite.

(W-3) If $R_1, R_2 \in W$, $U^{(1)}R_1 \cap m(R_2) \cap \bigcap_{R \in W} R = \emptyset$, then $R_1 = R_2$,

where $m(R)$ is the unique maximal ideal of a local ring R, and $U^{(i)}R = 1 + m(R)^i$ $(i \geqq 1)$. Then,

THEOREM 0. *Let K be a field. Then there exists an inclusion-reversing bijection between the set of all Dedekind domains A with quotient field K, and the set of all W satisfying the conditions* (W-0), (W-1), (W-2) *and* (W-3). *The bijection is defined by*

$$\begin{cases} A \longmapsto W : W \text{ is the set of all } P\text{-adic valuation rings defined by the maximal ideals } P \text{ of } A, \\ W \longmapsto A : A \text{ is the intersection of all valuation rings belonging to } W. \end{cases}$$

For a proof, see [3], Theorem 1.3, Theorem 1.4, and p. 441.

In this paper, we shall generalize Theorem 0 (see Theorems 9 and 13)

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