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On the Bernstein-Nikolsky Inequality

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1. Introduction.

It is well-known that while trigonometric polynomials are good means of approximation of periodic functions, entire functions of exponential type may serve as a mean of approximation of nonperiodic functions, given on n-dimensional space. Some properties of entire functions of exponential type, bounded on the real space $\boldsymbol{R}^n$ have been considered in [1]. These results are very important in the imbedding theory, the approximation theory and applications. The present paper is a continuation of this direction.

2. Results.

Let $1 \leq p \leq \infty$ and $\sigma = (\sigma_1, \cdots, \sigma_n)$, $\sigma_j > 0$, $j = 1, \cdots, n$. Denote by $M_{\sigma,p}$ the space of all entire functions of exponential type σ which as functions of a real $x \in \boldsymbol{R}^n$ belong to $L_p(\boldsymbol{R}^n)$. The well-known Bernstein-Nikolsky inequality reads as follows (see [1], p. 114): Let $f(x) \in M_{\sigma,p}$. Then

$$\sigma^{-\alpha} \| D^\alpha f \|_p \leq \| f \|_p , \qquad \alpha > 0 . \tag{1}$$

We have the following result:

THEOREM 1. *Given $1 \leq p < \infty$ and $f(x) \in M_{\sigma,p}$. Then*

$$\lim_{|\alpha| \to \infty} \sigma^{-\alpha} \| D^\alpha f \|_p = 0 . \tag{2}$$

To prove this theorem we need the following results:

LEMMA 1. *Let $0 < r \leq p \leq q \leq \infty$. Then $L_r(\boldsymbol{R}^n) \cap L_q(\boldsymbol{R}^n) \subset L_p(\boldsymbol{R}^n)$ and*

$$\| f \|_p \leq \| f \|_r^t \| f \|_q^{1-t}$$

for any $f(x) \in L_r(\boldsymbol{R}^n) \cap L_q(\boldsymbol{R}^n)$, where $t = (1/p - 1/q)/(1/r - 1/q)$.

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