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On the Galois Group of $x^p+ax+a=0$

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§1. Introduction.

Let p $(p>3)$ be a prime number, and let a be a rational integer with $(p, a)=1$ such that

$$f(x)=x^p+ax+a$$

is irreducible over the rational number field $\boldsymbol{Q}$. In the present paper we discuss the following problem: Is the Galois group of $f(x)=0$ over $\boldsymbol{Q}$ the symmetric group S_p? Our results will be stated in Theorem 1 and Theorem 2.

We require the following lemma of van der Waerden:

LEMMA 1 ([4]). *Let K be an algebraic number field of degree n, and let $\bar{K}$ denote the Galois closure of K over $\boldsymbol{Q}$. If the discriminant d of K is exactly divisible by a prime number q (i.e. $q \mid d$, $q^2 \nmid d$), then the Galois group of $\bar{K}/\boldsymbol{Q}$ contains a transposition (as a permutation group on $\{1, 2, \cdots, n\}$).*

§2. The case $p\equiv 3$ or 5 or 7 (mod 8).

THEOREM 1. *Let a denote a rational integer, and let p denote a prime number with the following properties:*

1. $p\equiv 3$ *or* 5 *or* 7 (mod 8), $p\neq 3$;
2. $(p, a)=1$;
3. $f(x)=x^p+ax+a$ *is irreducible over* $\boldsymbol{Q}$.

Then the Galois group of $f(x)=0$ over $\boldsymbol{Q}$ is the symmetric group S_p.

PROOF. Let α be a root of $f(x)=0$, and let $K=\boldsymbol{Q}(\alpha)$, $\delta=f'(\alpha)$, $D=$norm δ (in K). Then ([1], Theorem 2)

$$D=a^{p-1}\{(p-1)^{p-1}a+p^p\} .$$

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