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On the Existence and Uniqueness of the Stationary Solution to the Equations of Natural Convection

Hiroko MORIMOTO

Meiji University

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1. Notations and results.

In this paper, we discuss the existence of weak solutions of a system of equations which describes the motion of fluid with natural convection (Boussinesq approximation) in a bounded domain Ω in $\boldsymbol{R}^n$, $2 \leq n$. We consider the following system of differential equations:

$$\begin{cases} (u\cdot\nabla)u = -\dfrac{1}{\rho}\nabla p + \nu\Delta u + \beta g\theta , & \\ \operatorname{div} u = 0 , & \text{in} \quad \Omega \\ (u\cdot\nabla)\theta = \chi\Delta\theta , & \end{cases} \tag{1}$$

where $u\cdot\nabla = \sum_j u_j \partial/\partial x_j$. Here u is the fluid velocity, p is the pressure, θ is the temperature, g is the gravitational vector function, and ρ (density), ν (kinematic viscosity), β (coefficient of volume expansion), χ (thermal diffusivity) are positive constants. We study this system of equations with mixed boundary condition for θ. Let $\partial\Omega$ (the boundary of Ω) be divided into two parts Γ_1, Γ_2 such that

$$\partial\Omega = \Gamma_1 \cup \Gamma_2 , \qquad \Gamma_1 \cap \Gamma_2 = \varnothing .$$

The boundary conditions are as follows.

$$\begin{cases} u=0 , \quad \theta = \xi , & \text{on} \quad \Gamma_1 , \\ u=0 , \quad \dfrac{\partial\theta}{\partial n} = \eta , & \text{on} \quad \Gamma_2 , \end{cases} \tag{2}$$

where ξ (resp. η) is a given function on Γ_1 (resp. Γ_2), n is the outward normal vector to $\partial\Omega$.

In this paper, we show the existence of weak solution of this problem for bounded domain Ω in $\boldsymbol{R}^n$, $2 \leq n$, using the Galerkin method (Theorem 1). Some uniqueness result

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